

Sections 8.1, 8.2, 8.3 — Estimating Population Means

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Outline

σ is known

Student's t -distribution

σ is Unknown

σ is known

Margin of error

If the population standard deviation is known, the margin of error is:

$$E = (z_{\alpha/2}) \left(\frac{\sigma}{\sqrt{n}} \right)$$

if the following requirements are met:

1. The sample is a simple random sample.
2. The sample size is at least 30 or the population distribution is normally distributed.

Home Maintenance

A survey of 85 randomly selected homeowners finds that they spend a mean of \$67 per month on home maintenance. Construct and interpret a 99% confidence interval for the mean amount of money spent per month on home maintenance by all home owners. Assume that the population standard deviation is \$14 per month.

Minimum Sample Size

The minimum sample size required for estimating a population mean at a given level of confidence with a particular margin of error is

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

Credit Cards

A bank would like to estimate the mean number of credit cards college students have in their wallets. They would like to create a 98% confidence interval with a maximum error of 1 card. Assuming a standard deviation of 3.25 cards, what is the minimum number of college students they must include in their sample?

Student's t -distribution

Properties

1. A t -distribution is symmetric and bell-shaped, centered around 0.
2. A t -distribution is completely defined by its number of *degrees of freedom*, df .
3. The total area under its graph is 1.
4. The x -axis is a horizontal asymptote for a t -distribution.

Examples

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2. Find $t_{0.050}$ if $df = 50$
3. Find $t_{0.005}$ if $df = 150$
4. Find the value of t so the t -distribution with 11 degrees of freedom has an area to the left of 0.010.
5. Find the value of t so that the area between t and $-t$ is 0.95

σ is Unknown

Margin of Error

If the population standard deviation is not known, the margin of error is:

$$E = (t_{\alpha/2}) \left(\frac{s}{\sqrt{n}} \right)$$

with

$$df = n - 1$$

if the following requirements are met

1. The sample is a simple random sample.
2. The sample size is at least 30 or the population is normally distributed.

Atkins Diet

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1. Construct a 90% confidence interval estimate of the mean weight loss for all such subjects.
2. What would it mean (for \bar{x}) for the Atkins diet to work?
3. Based solely off of this data, do you think the Atkins diet works? Is it worth it? What other information may we want?