

## Section 5.3 – Poisson Distribution

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Introduction

Examples

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1. The random variable  $X$  is the number of occurrences of an event over some interval.
2. Each success must be independent of other successes.
3. The mean number of successes in a given interval must be constant



# Probability and Parameters

## Probability

For a Poisson random variable  $X$ , the probability of obtaining exactly  $x$  successes in any particular interval is given by

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

where  $e \approx 2.718282$  and  $\lambda$  is the mean number of successes.

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- The mean is  $\mu = \lambda$ .
- The standard deviation of  $\sigma = \sqrt{\lambda}$ .

## Examples

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- What is the probability that they receive fewer than 10?

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- What is the probability that I will kill 7 spiders in the next three weeks?
- What is the probability that I will kill no more than 4 in the next three weeks?

# Horse kicks!

The number of deaths caused by horse kicks to men in the Prussian Army between 1875 and 1894 can be modeled with a Poisson distribution. Over 20 years and 14 corps, there were 196 deaths in 280 corps-years. `#+ATTR_BEAMER :overlay +(1)-`

- What is the probability that a randomly selected corps-year has 0 deaths?
- What about 1? 2? 3? 4?

## Actual numbers for deaths by horse-kick.

The actual numbers were

Number of deaths	Number of corps-years
0	144
1	91
2	32
3	11
4	2