

# Section 5.1 — Discrete Random Variables

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# Outline

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MOAR EXAMPLES!!!

# Introduction

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# Definitions

## Definition (Random Variable)

A **random variable** is a variable (typically  $X$ ,  $Y$ , or  $Z$ ) that has a single numerical value, determined by chance, for each outcome of a procedure.

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## Definition (Probability Distribution)

A **probability distribution** is a description that gives the probability for each value of the random variable. It is often expressed in the format of a table, formula, or graph.

# Random variables

## Definition (Discrete Random Variable)

A **discrete random variable** has a collection of values that is finite or determined by a counting process.

# Requirements of a Distribution

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2.  $0 \leq P(X = x) \leq 1$  for every individual value of the random variable  $x$ .
3.  $\sum P(X = x) = 1$

## Examples

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# Marijuana Legalization

**Table 1:** Responses to the question "Should marijuana use be legal?"

Response	$P(X = x)$
Yes	0.409
No	0.520
Don't Know	0.070

Is this a probability distribution?

# Salary Discussion

**Table 2:** Responses to the question "On which interview should a candidate begin salary negotiations?"

Number of Interviews $x$	$P(X = x)$
1	0.30
2	0.26
3	0.10

Is this a probability distribution?

$$P(X = x) = \frac{x}{10} \text{ for } x = 0, 1, 2, 3, 4$$

Is this a probability distribution?

# Parameters

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# Expected Value

The **expected value** for a discrete random variable  $X$  is equal to the mean of the probability distribution. It is given by

$$E(X) = \mu = \sum (x \cdot P(X = x))$$

## Definition (Variance)

The variance of a probability distribution is either

$$\sigma^2 = \sum \left( (x - \mu)^2 \cdot P(X = x) \right)$$

or

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## Standard Deviation

The standard deviation of a probability distribution is

$$\sigma = \sqrt{\sum (x^2 \cdot P(X = x)) - \mu^2}$$

**MOAR EXAMPLES!!!**

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# Genetic Disorders

Four males with an X-linked genetic disorder have one child each. The random variable  $x$  is the number of children among the four who inherit the genetic disorder.

**Table 3:** Number of children among with disorder

$x$	$P(X = x)$
0	0.0625
1	0.2500
2	0.3750
3	0.2500

## Texas Pick 3

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- What is the probability of winning?
- What is the expected value of your winnings?