

# Section 3.3 – Measures of Relative Position

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# Outline

Percentiles

Box Plots

z Scores

Outliers

# Percentiles

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# Percentiles

## Definition (Percentiles)

**Percentiles** are measures of location, denoted  $P_1, P_2, \dots, P_{99}$  which divide a set of data into 100 groups each with about 1% of the values in it.

## Find the Percentile of A Data Value

$$\text{Percentile of } x = \frac{\text{number of values less than or equal to } x}{\text{total number of values}} \cdot 100$$

# Finding the Data Value of a Percentile

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3. If  $L$  is a whole number,  $P_k$  is the number midway between the  $L^{\text{th}}$  value and the one after it.
4. If  $L$  is not a whole number, round up and  $P_k$  is the number in the  $L^{\text{th}}$  position.

## Definition (Quartiles)

- The **first quartile**,  $Q_1$ , is  $P_{25}$  and separates the bottom 25% from the top 75%.
- The **second quartile**,  $Q_2$ , is the median.
- The **third quartile**,  $Q_3$ , is  $P_{75}$  and separates the bottom 75% from the top 25%.

# Five number summary

## Definition (Five number summary)

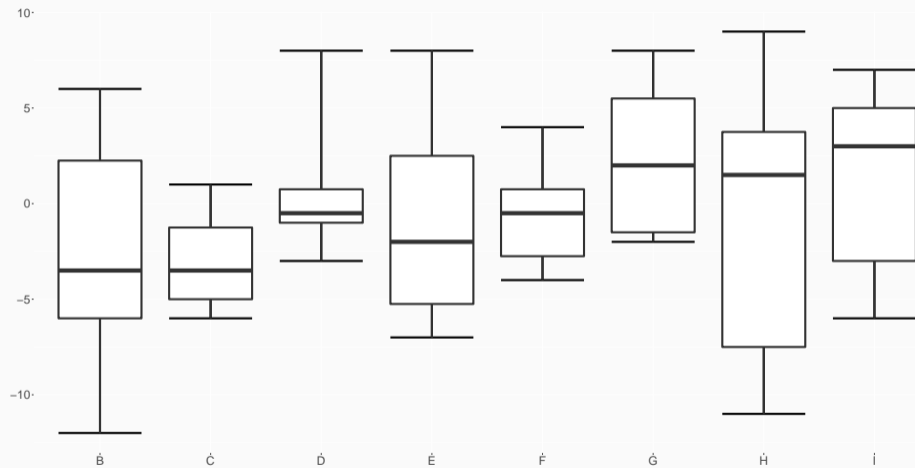
The five number summary consists of the following:

1. Minimum
2.  $Q_1$
3. Median
4.  $Q_3$
5. Maximum

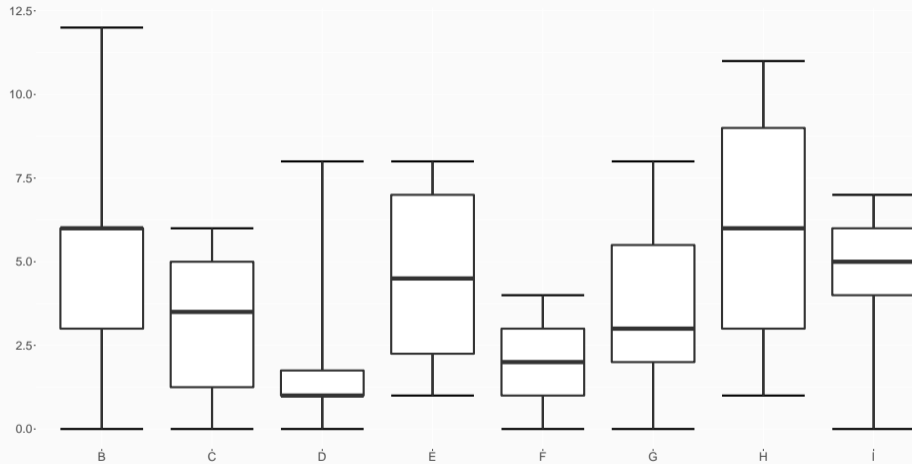
# Box Plots

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# Age Guessing



# Age Guessing (absolute value)



## Who's cooler?

You and your friend are trying to determine who is the cooler one. Obviously you're going to compare standardized test scores. The problem is you took the SAT your friend took the ACT. You got a 1261 on the SAT and your friend got a 27 on the ACT. Use the info below to determine who's cooler.

	SAT	ACT
$\bar{x}$	1060.0	21.0
$s$	195.0	5.1

# z Scores

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## Definition (z score)

A **z score** (also called a **standard score**) is the number of standard deviations that given value  $x$  is above or below the mean.

$$z = \frac{x - \bar{x}}{s}, z = \frac{x - \mu}{\sigma}$$

# Age and Shoe Size

Am I older than my feet are bigger?

Age	Shoe Size
$\bar{x}$	
s	

# Curving Scores

I have decided on the following curve for my class

**Table 1:** z-score cutoffs

Letter	Minimum z score	Meaning
A	1.29	Top 10%
B	0.68	Top 25%
C	-0.67	Middle 50%
D	-1.28	Top 90%

# Class scores

One of my classes (from a different school) had the following grades

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51	71	58	91	95	66	53
85	72	100	89	70	43	83
75	82	79	58	71	74	92

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# Curving Scores

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Should I curve your scores?

# Outliers

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The **interquartile range** (or **IQR**) is  $Q_3 - Q_1$

## $1.5 \times IQR$

A value is a *potential* outlier if it is

- larger than  $Q_3 + 1.5 \times IQR$  **or**
- less than  $Q_1 - 1.5 \times IQR$